PROJECTILE MOTION PROBLEMS
full solutions

These solutions build on the skills covered in my video series "Vector components".

Step-by-step discussions for each of these solutions are available in the "Projectile motion problems" videos.
The problems are available in the Problems document.
Answers without solutions are available in the Answers document.
You can find links to these resources at my website: www.freelance-teacher.com

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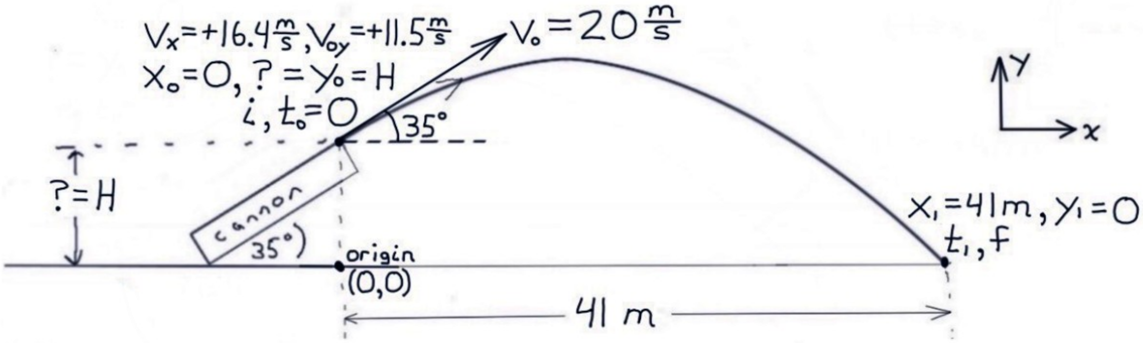
If you find that the video explanations move too slowly, you can simply try the problems in the Problems document, study the solutions in the Solutions documents, and, if necessary, skip to the particular parts of the videos that cover the parts of the solutions that are giving you difficulty. Each video has a table of contents, to make it easier to skip to particular topics.

If you find a particular problem to be difficult, then, after studying the solution, *before* you try the next problem, you should take a blank piece of paper and retry that problem from scratch. Don't move on to the next problem in the series until you are comfortable with the solution for the current problem.

Video (1)

A cannon shoots a ball with initial velocity 20.0 m/s at an angle of 35.0° upward from the horizontal. The ball lands at a horizontal distance of 41.0 m from the cannon. What is the height *H* of the top of the cannon barrel above the ground?



$x_f = x_i + v_x \Delta t$

$41 = 0 + 16.4\,\Delta t$

$41 = 16.4\,\Delta t$

$\frac{41}{16.4} = \frac{\cancel{16.4}\,\Delta t}{\cancel{16.4}}$

$\Delta t = 2.5\text{ s}$

?

$\Delta t,\ y_i,\ y_f,\ v_{iy},\ v_{fy},\ a_y$

$\Delta t,\ H,\ 0,\ +11.5\,\frac{m}{s},\ V_{1y},\ -9.8\,\frac{m}{s^2}$

2.5 s

$y_f = y_i + v_{iy}\Delta t + \frac{1}{2} a_y (\Delta t)^2$

$0 = H + 11.5(2.5) + \frac{1}{2}(-9.8)(2.5)^2$

$0 = H - 1.88$

$+1.88 \qquad +1.88$

$1.88\text{ m} = H$

Answer: $\boxed{H = 1.9\text{ m}}$

For a projectile motion problem, build as much information as you can into your sketch, as shown above. If possible, build the question into the sketch, as shown above.

See next page for how to break the initial velocity vector into components.

The key to succeeding with projectile motion problems is to create a projectile motion "setup" for each problem. The setup for this problem is illustrated above.

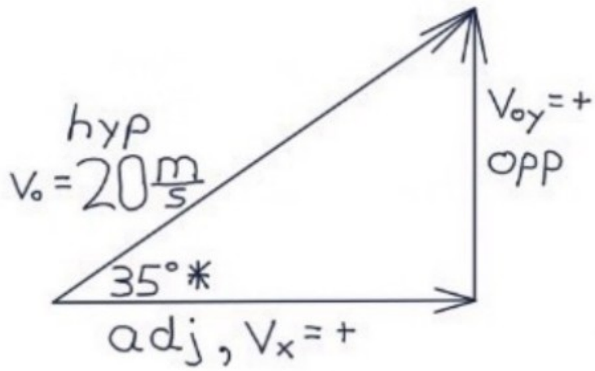
In projectile motion, the vertical velocity is changing, but the *horizontal* velocity is *constant*. Because the horizontal velocity is constant, the setup for the horizontal component consists of writing the equation $x_f = x_i + v_x \Delta t$.

Because the vertical velocity is changing, the setup for the vertical component consists of writing the six kinematics variables for the y-component: Δt, y_i, y_f, v_{iy}, v_{fy}, a_y.

Use the information from your sketch to substitute specific values and symbols into your projectile motion setup, as shown above.

As illustrated above, arrange your work for the x-component, and your work for the y-component, in *two adjacent columns*. This will help you to keep your math *organized*.

Here is the process for breaking the initial velocity vector into components.



SOH CAH TOA

$$\sin 35° = \frac{opp}{hyp}$$
$$\sin 35° = \frac{|V_{0y}|}{20}$$
$$20 \cdot \sin 35° = \frac{|V_{0y}|}{\cancel{20}} \cdot \cancel{20}$$
$$|V_{0y}| = 11.5 \frac{m}{s}$$
$$V_{0y} = +11.5 \frac{m}{s}$$

$$\cos 35° = \frac{adj}{hyp}$$
$$\cos 35° = \frac{|V_x|}{20}$$
$$20 \cos 35° = \frac{|V_x|}{\cancel{20}} \cdot \cancel{20}$$
$$|V_x| = 16.4 \frac{m}{s}$$
$$V_x = +16.4 \frac{m}{s}$$

In projectile motion, the vertical velocity is changing, but the *horizontal* velocity is *constant*.

Since the horizontal velocity is constant, we can use the single symbol v_x to represent the horizontal velocity at any point during the projectile motion. Since the vertical velocity is changing, we use the symbol v_{0y} to represent the vertical velocity specifically at time t_0.

We use absolute value symbols in our SOH CAH TOA equations because the SOH CAH TOA equations can only tell us the *magnitudes* of the components. We determine the *signs* of the components ("+" or "-") in a separate step, based on the directions of the component arrows in our right triangle.

I recommend that you should include a plus sign in front of positive components (e.g., on this problem, v_x = +11.5 m/s and v_{0y} = +16.4 m/s), because that will help you to remember to include the crucial negative signs in front of negative components (e.g., on this problem, a_y = -9.8 m/s^2).

Make sure you understand the SOH CAH TOA process we used above for breaking the initial velocity into components. I have another video series on "Vector Components" that focuses on the process for breaking vectors into components.

Do our results make sense?

$\Delta t = 2.5\ \mathrm{s}$, $H = 1.88\ \mathrm{m}$

Does it make sense that our result for Δt is positive?
Δt stands for the time elapsed. Time elapsed can never be negative, so, yes, it does make sense that our result for Δt is positive.

Does it make sense that our result for H is positive?
H stands for the cannonball's height at time t_0. It wouldn't make sense for the height to be negative; so, yes, it does make sense that our result for H is positive.

Does the magnitude of our result for H make sense?
If you're more used to using yards and feet in everyday life, rather than meters, then it will be helpful to know that 1 meter is roughly 1 yard; and 1 yard is 3 feet.
Therefore 1.88 m is roughly 2 m, which is roughly 2 yards. I would say that yes, a height of 2 yards, or 6 feet, for a cannon does make sense.
The cannonball travels a horizontal distance of about forty yards (almost half a football field), in two and a half seconds, before it hits the ground.

The cannonball is shot with an initial speed of 20 m/s. 1 meter per second is, very roughly, 2 miles per hour. So, the cannonball was shot with a fairly low initial speed of roughly 40 mi/hr.
(If you live in a country where driving speeds are measured in kilometers per hour, it helps to know that 1 m/s is, very roughly, 4 kilometers per hour; so the cannonball's initial speed is roughly 80 km/hr.)

Recap

The solution for this problem demonstrates a systematic method for solving projectile motion problems: (1) Build as much kinematics information as possible into your sketch; (2) use the information from your sketch to create a projectile motion "setup" for the problem; (3) use the setup to solve the problem.

To succeed with projectile motion problems, think in terms of *components*.

The problem told us that the cannonball's initial velocity is 20 m/s. So let me ask you: where did we plug the number 20 m/s into our kinematics equations?

Answer: We did *not* plug 20 m/s into our kinematics equations, because 20 m/s represents the magnitude of the *overall* velocity vector. Instead, we broke the 20 m/s into *components*, and then we substituted each of those *components* (v_x and v_{0y}) into the kinematics equations.

In projectile motion, the vertical velocity is changing, but the horizontal velocity is *constant*. So, for the horizontal velocity, we use the *single* symbol v_x; but, for the vertical velocity, we need two symbols, v_{iy} and v_{fy}.

And, we need to use a different "setup" for the horizontal component of projectile motion, than for the vertical component of projectile motion. For the horizontal component, we write the single equation $x_f = x_i + v_x\,\Delta t$. For the vertical component, the setup consists of writing the six kinematics variables for the y-component.

$$x_f = x_i + v_x\,\Delta t \qquad \Delta t,\ \ y_i,\ \ y_f,\ \ v_{iy},\ \ v_{fy},\ \ \begin{array}{c} a_y \\ -9.8\,\frac{\text{m}}{\text{s}^2} \text{ or } -g \end{array}$$

Underneath the general variables in your setup, write the *specific* numbers or symbols that apply for each variable for the particular problem you're working on. The projectile motion setup helps us to *organize* the kinematics data that we will need to solve the problem. **The key to succeeding with projectile motion problems is to create this projectile motion setup for each problem.**

Projectile motion occurs when the only force on the object is the force of the Earth's gravity. In projectile motion, the magnitude of the acceleration is 9.8 m/s^2, the magnitude of the acceleration due to the Earth's gravity. Since gravity pulls down, the acceleration during projectile motion always points *down*; so, if you choose *up* as your positive y-direction, for projectile motion you will have that $a_y = -9.8$ m/s^2. Don't forget the crucial negative sign!

For a projectile motion problem, the *connecting link* between the x-component and the y-component is Δt. To solve the problem, we expect to use one component to find Δt, then substitute our result for Δt into the framework for the other component. For example, for this problem, we used the x-component to find Δt; then we substituted our result for Δt into the setup for the y-component. Arrange your work for the x-component and y-component in *two adjacent columns*; this helps to keep the math organized.

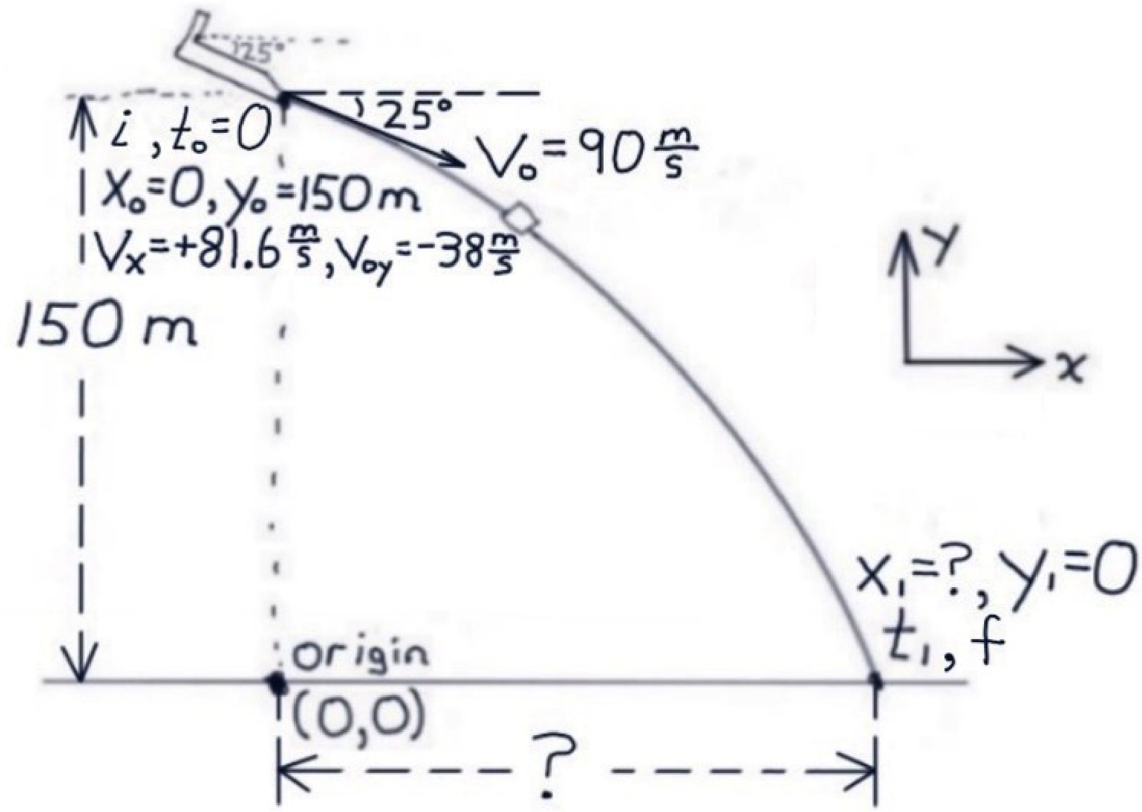
Build as much information as possible into your sketch. If you can, build the question into the sketch. Draw a *large* sketch, so that there will be ample room to clearly include all relevant information in the sketch.

Video (2)

Part (a)

An airplane releases a package while diving at a downward angle of 25° below the horizontal. The plane is traveling at a height of 150 m above the ground, with speed 90 m/s, when the package is released.
(a) How far away horizontally from the release point will the package land on the ground?
(b) What is the package's speed when it hits the ground?

Build as much information as possible into your sketch, as illustrated below. Draw a *large* sketch, so there will be ample room to include all pertinent information.



Use the information you've built into your sketch to create a projectile motion "setup", as shown below. **The key to succeeding with projectile motion problems is to write down the projectile motion setup**.

$$\overset{?}{X_f} = x_i + V_x \Delta t \qquad \Delta t,\ y_i,\ y_f,\ V_{iy},\ V_{fy},\ a_y$$
$$X_1 = 0 + 81.6\,\Delta t \qquad \Delta t,\ 150\text{m},\ 0,\ -38\tfrac{m}{s},\ V_{1y},\ -9.8\tfrac{m}{s^2}$$

On the next page, we show how to use this "setup" to solve part (a).

$$\overset{?}{x_f} = x_i + v_x \Delta t \qquad \Delta t,\ y_i,\ y_f,\ v_{iy},\ v_{fy},\ a_y$$

$$x_1 = 0 + 81.6\Delta t \qquad \Delta t,\ 150\text{m},\ 0,\ -38\tfrac{\text{m}}{\text{s}},\ v_{1y},\ -9.8\tfrac{\text{m}}{\text{s}^2}$$

$$x_1 = 81.6(2.88)$$

$$x_1 = 235\text{ m}$$

$$y_f = y_i + v_{iy}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

$$0 = 150 + (-38)\Delta t + \tfrac{1}{2}(-9.8)(\Delta t)^2$$

$$0 = 150 + (-38)\Delta t + (-4.9)(\Delta t)^2$$

$$a(\Delta t)^2 + b\Delta t + c = 0$$

$$-4.9(\Delta t)^2 + (-38)\Delta t + 150 = 0$$

$$a = -4.9,\ b = -38,\ c = 150$$

$$\Delta t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Delta t = \frac{-(-38) \pm \sqrt{(-38)^2 - 4(-4.9)(150)}}{2(-4.9)}$$

$$\Delta t = \frac{38 \pm \sqrt{38^2 + 4(4.9)(150)}}{2(-4.9)}$$

$$\Delta t = \cancel{-10.6\text{ s}},\ 2.88\text{ s}$$

We have used the quadratic formula to find Δt. Notice the process that we used to determine values for *a*, *b*, and c. When using the quadratic formula, be careful to include all necessary negative signs.

The symbol ± stands for "plus or minus", so we perform the calculation once as an addition and once as a subtraction. You will usually need to throw out one of your results. Throw out the result that doesn't make sense in the context of the problem.

The video explanation describes how you can evaluate the expression for the quadratic formula in one step on your calculator.

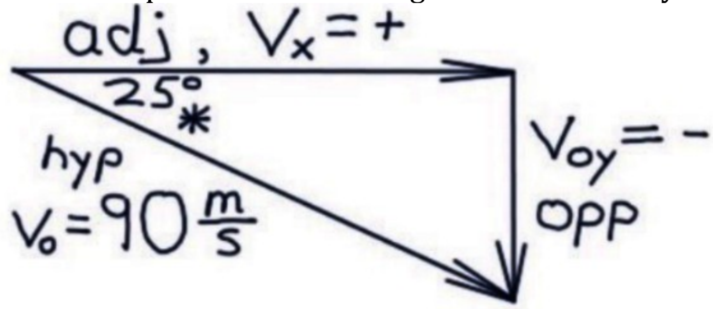
See next page for the process for breaking the initial velocity into components.

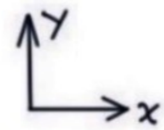
Answer for part (a)

The package will land at a horizontal distance of 235m from the release point.

Be sure to include *units* in your answer. An answer without units is *wrong*.

Here is the process for breaking the initial velocity into components.





SOH CAH TOA

$$\sin 25° = \frac{opp}{hyp}$$

$$\sin 25° = \frac{|V_{oy}|}{90}$$

$$90 \cdot \sin 25° = \frac{|V_{oy}|}{\cancel{90}} \cdot \cancel{90}$$

$$|V_{oy}| = 38 \frac{m}{s}$$

$$V_{oy} = -38 \frac{m}{s}$$

$$\cos 25° = \frac{adj}{hyp}$$

$$\cos 25° = \frac{|V_x|}{90}$$

$$90 \cdot \cos 25° = \frac{|V_x|}{\cancel{90}} \cdot \cancel{90}$$

$$|V_x| = 81.6 \frac{m}{s}$$

$$V_x = +81.6 \frac{m}{s}$$

We use absolute value symbols in our SOH CAH TOA equations because the SOH CAH TOA equations only tell us the *magnitudes* of the components. We determine the *signs* of the components ("+" or "-") in a separate step, based on the directions of the component arrows in our right triangle.

It is *crucial* to include the negative sign in front of v_{0y}!

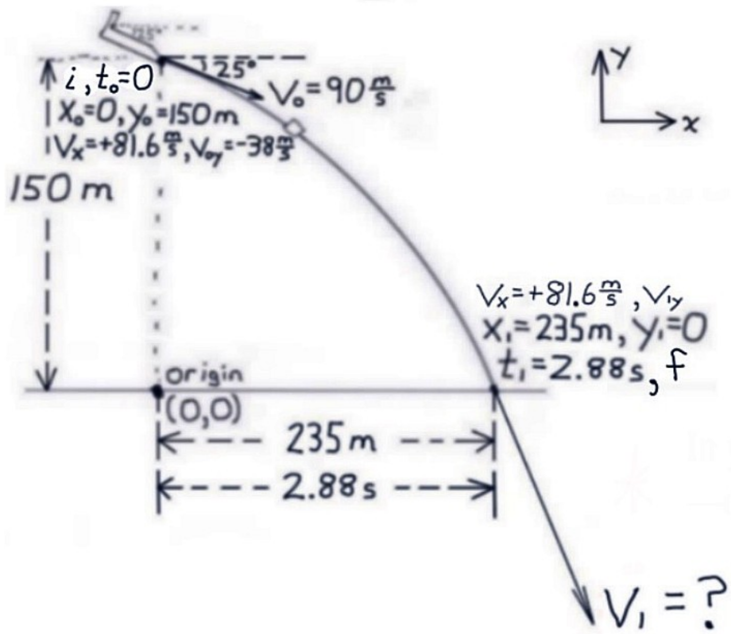
Include a plus sign in front of positive components (for this problem, $v_x = +81.6$ m/s), because that will help you to remember to include the crucial negative signs in front of negative components (for this problem, $v_{0y} = -38$ m/s, and $a_y = -9.8$ m/s^2).

In projectile motion, the vertical velocity is changing, but the *horizontal* velocity is *constant*.

Since the horizontal velocity is constant, we can use the single symbol v_x to represent the horizontal velocity at any point during the projectile motion. Since the vertical velocity is changing, we use the symbol v_{0y} to represent the vertical velocity specifically at time t_0.

Part (b)

(b) What is the package's speed when it hits the ground?



In projectile motion, the horizontal velocity is constant, while the vertical velocity is changing. So the horizontal velocity at t_1 is the same as the horizontal velocity at t_0 (+81.6 m/s); and the vertical velocity at t_1 is different from the vertical velocity at t_0. To find the vertical velocity at t_1 (v_{1y}), we use another kinematics equation, as shown below; and we use our result for Δt (2.88 s) from part (a).

need

$$\Delta t,\ y_i,\ y_f,\ V_{iy},\ V_{fy},\ a_y$$

$$\Delta t,\ 150\text{m},\ 0,\ -38\tfrac{m}{s},\ V_{1y},\ -9.8\tfrac{m}{s^2}$$

$$x_f = x_i + V_x \Delta t$$

$$X_1 = 0 + 81.6\,\Delta t$$

$$X_1 = 81.6\,(2.88)$$

$$X_1 = 235\text{ m}$$

2.88 s

$$y_f = y_i + V_{iy}\,\Delta t + \tfrac{1}{2} a_y (\Delta t)^2$$

$$0 = 150 + (-38)\Delta t + \tfrac{1}{2}(-9.8)(\Delta t)^2$$

$$0 = 150 + (-38)\Delta t + (-4.9)(\Delta t)^2$$

$$\Delta t = \cancel{-10.6}\text{ s},\ 2.88\text{ s}$$

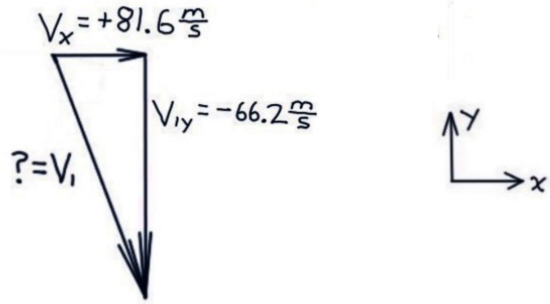
$$V_{fy} = V_{iy} + a_y(\Delta t)$$

$$V_{fy} = -38 + (-9.8)(2.88)$$

$$V_{fy} = -66.2\ \tfrac{m}{s}$$

Does it make sense that our result for v_{1y} is negative? The negative y-direction is *down*, so our negative result indicates that the package's vertical motion at t_1 is downward. Based on the path of motion, we expected that the package's vertical motion at t_1 would be downward. So, yes, it does make sense that our result for v_{1y} is negative.

Now that we know the *components* of the velocity at t_1, we can use the Pythagorean theorem to find the magnitude of the *overall* velocity vector at t_1.



Pythagorean theorem

$$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{other leg})^2$$

$$V_1^2 = 81.6^2 + 66.2^2$$

$$V_1^2 = 11041$$

$$V_1 = \sqrt{11041}$$

$$V_1 = 105 \frac{m}{s}$$

Answer for part (b)

$\boxed{\text{The package hits the ground with a speed of } 105 \frac{m}{s}}$

Be sure to include *units* in your answer. An answer without units is *wrong*.

The table below shows the kinematics equations we use for projectile motion problems.

the kinematics equations for two-dimensional projectile motion

***x* equation** (constant v_x)	***y* equations** (constant a_y, changing v_y)	**missing variables**
$x_f = x_i + v_x \Delta t$	$y_f = y_i + v_{iy} \Delta t + \frac{1}{2} a_y (\Delta t)^2$	v_{fy}
	$v_{fy} = v_{iy} + a_y \Delta t$	y_i, y_f
	$v_{fy}^2 = v_{iy}^2 + 2a_y (y_f - y_i)$	Δt

Do our results make sense?

Does it make sense that our result for x_1 is positive? x_1 stands for the x-coordinate of the position at time t_1. Our sketch shows that, at time t_1, the package is located *rightwards* from the origin. We chose *right* as our positive x-direction, so, yes, it does make sense that our result for x_1 is positive.

Does the magnitude of our result for x_1 make sense?
Well, the *horizontal* distance of 235 meters is comparable in size to the initial *vertical* height of 150 meters, so yes, the magnitude of our result for x_1 is reasonable.
Remember that 1 meter is roughly 1 yard. And, for purposes of comparison, keep in mind that a football field is 100 yards long. So, 235 meters is roughly 235 yards, which is more than two football fields. 150 meters is roughly one and a half football fields. So, the package was released from a height of about one a half football fields, and traveled a horizontal distance of more than two football fields before hitting the ground.

Does the magnitude of our result for v_{1y} make sense?
It's interesting to compare the magnitude of v_{1y} (66.2 m/s) with the magnitude of v_{0y} (38 m/s). Does it make sense that the magnitude of v_{1y} is bigger than the magnitude of v_{0y}?
The package experiences a *downward* pull from gravity. Therefore, as the package falls from the plane, we expect that the package's vertical speed will *increase*. So, yes, it does make sense that the package's vertical speed when it hits the ground will be greater than the package's vertical speed when it is released from the airplane.

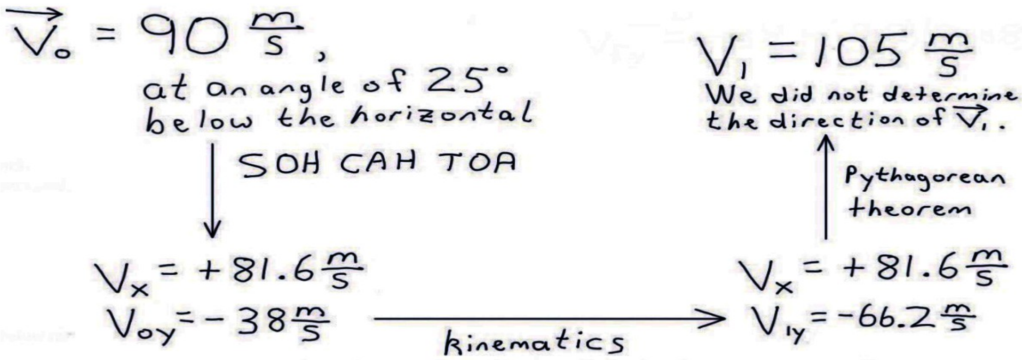
The package's vertical speed is increasing. In contrast, notice that the *horizontal* speed (81.6 m/s) is the same when the package hits the ground as when it was released from the plane. Since the package's vertical speed is increasing, it makes sense that *overall* speed when the package hits the ground (105 m/s) is greater than the overall speed when it's released from the plane (90 m/s).

The plane is moving at 90 m/s. 1 m/s is, very roughly, 2 miles per hour, so the plane is moving at roughly 180 mi/hr when the package is released. That's a low speed for an airliner, but a fast speed for a small, single-engine plane.
(If you live in a country where driving speeds are measured in km/hr, it will useful to know that 1 m/s is, very roughly, 4 km/hr; so, 90 m/s is roughly 360 km/hr.)

Our result for Δt was 2.88 s. This tells us that it takes less than 3 s for the package to fall a vertical distance of roughly one and half football fields. This quick fall time is partly because the initial vertical speed is 38 m/s (roughly 40 yards per second). If the package were dropped from the same height with zero initial vertical speed, than it would take about 5.5 s to hit the ground.

Recap

In this problem, we had to use the quadratic formula. For extra practice, **try it again**: use the quadratic formula to find the value of Δt based on the equation "$0 = 150 - 38\Delta t - 4.9(\Delta t)^2$". You should again get the result that Δt equals 2.88 s.



Think in terms of *components*. The problem gives us information about the *overall* velocity vector at t_0. And part (b) asks for the magnitude of the overall velocity vector at t_1. But, we did *not* figure out the velocity vector at t_1 directly from the velocity vector at t_0. Instead, we broke the velocity vector at t_0 into *components*. Then, we used the velocity *components* at t_0 to determine the velocity *components* at t_1. Then, we used the velocity components at t_1 to determine the magnitude of the *overall* velocity vector at t_1.

It's *crucial* to include the negative sign on v_{0y} (v_{0y} = -66.2 m/s). Include a plus sign in front of all your positive components (e.g., v_x = +81.6 m/s), because that will help you to notice when you need a negative sign in front of negative components, like v_{0y}.

In projectile motion, the vertical velocity is changing, but the horizontal velocity is *constant*. So the horizontal velocity at t_1 is the same as the horizontal velocity at t_0. You should use the same symbol for things that are equal, so we used the same symbol, v_x, for the horizontal velocity at t_0 and for the horizontal velocity at t_1.

In contrast, the vertical velocity is changing, so we had to use a kinematics equation to determine the vertical velocity at t_1. You should use different symbols for things that are unequal, so we used the symbol v_{0y} for the vertical velocity at t_0, and a *different* symbol, v_{1y}, for the vertical velocity at t_1.

If you know the *components* of a vector, you can use the Pythagorean theorem to find the magnitude of the overall vector, as we illustrated in our solution.

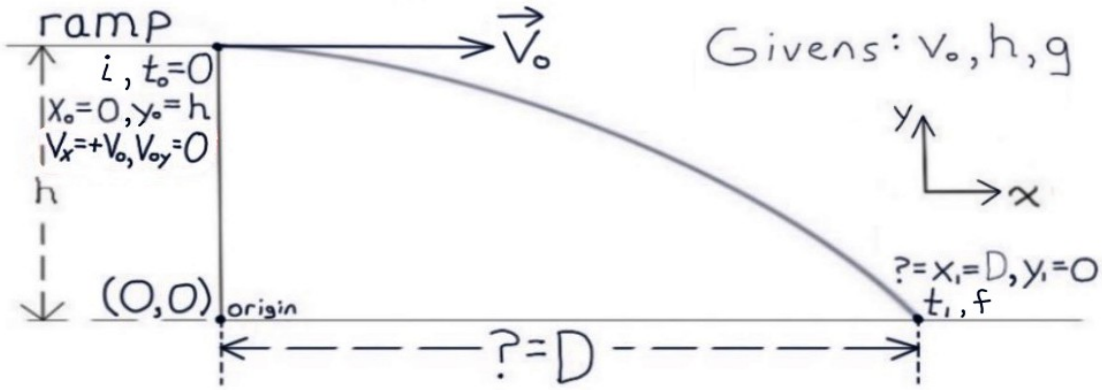
For projectile motion, the *connecting link* between the x-component and the y-component is Δt. In this problem, we used the y-component to determine Δt, then plugged our result for Δt into the equation for the x-component. (This is the reverse of the process we used for the problem in the previous video.)

After we used the y-component to determine Δt, we also built that information back into our setup for the y-component. Then, we used the setup for the y-component a *second* time. The second time, we used the setup for the y-component to find v_{fy}.

Arrange your work for the x-component, and your work for the y-component, in two adjacent *columns*. That will help you to keep your math *organized*.

Video (3)

A stunt motorcyclist leaves a horizontal ramp at speed v_0. The ramp is at a height of h above the ground. What horizontal distance D from the ramp does the motorcycle travel before it hits the ground?



$$\overset{?}{x_f} = x_i + v_x \Delta t, \qquad \Delta t, y_i, y_f, v_{iy}, v_{fy}, a_y$$

$$D = 0 + v_0 \Delta t \qquad \Delta t, h, 0, 0, v_{1y}, -g$$

$$D = v_0 \Delta t$$

$$D = v_0 \sqrt{\frac{2h}{g}}$$

$$y_f = y_i + v_{iy} \Delta t + \tfrac{1}{2} a_y (\Delta t)^2$$

$$0 = h + 0\,\Delta t + \tfrac{1}{2}(-g)(\Delta t)^2$$

$$0 = h - \tfrac{1}{2} g (\Delta t)^2$$

$$+\tfrac{1}{2} g (\Delta t)^2 \qquad +\tfrac{1}{2} g (\Delta t)^2$$

$$\tfrac{1}{2} g (\Delta t)^2 = h$$

$$\frac{1}{2} \cdot \frac{g}{1} (\Delta t)^2 = h$$

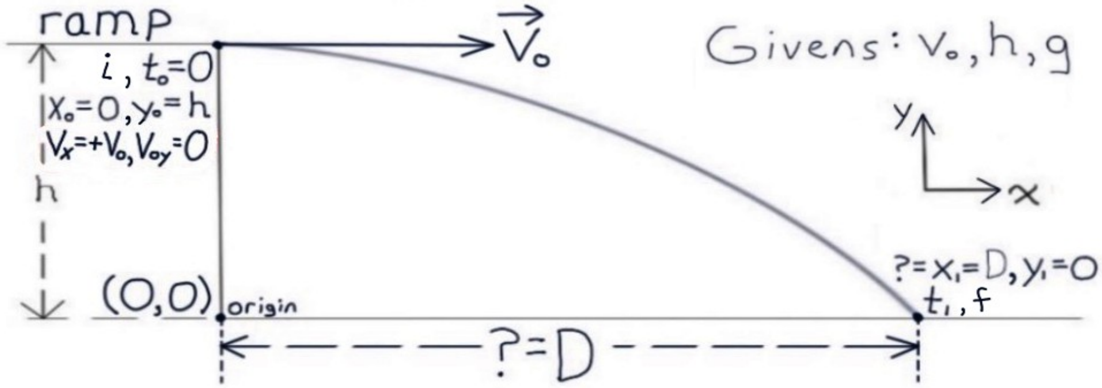
$$\frac{\cancel{2}}{1} \cdot \frac{1}{\cancel{g}} \cdot \frac{1}{\cancel{2}} \cdot \frac{\cancel{g}}{1} (\Delta t)^2 = \frac{h}{1} \cdot \frac{2}{1} \cdot \frac{1}{g}$$

$$(\Delta t)^2 = \frac{2h}{g}$$

$$\Delta t = \sqrt{\frac{2h}{g}}$$

On the next page we discuss how we broke the initial velocity into components.

Draw the object's *path of motion*. The path for two-dimensional projectile motion is a *parabola*. Since the ramp is horizontal, the slope of the parabolic path at time t_0 should be *horizontal*. The slope of the parabola at time t_0 should *not* be upward or downward. The slope of the parabola immediately *after* time t_0 should be downward.



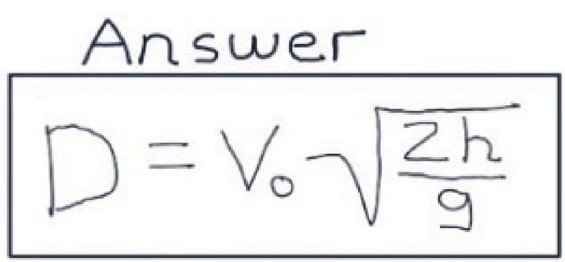
At time t_0, the motorcycle is leaving a *horizontal* ramp.
Therefore, the slope of the parabolic path at time t_0 is *horizontal*.
Therefore, **the velocity vector at time t_0 is *horizontal***.

If a vector is parallel or anti-parallel to one of the axes, then:
the component for that axis has the same magnitude and direction as the overall vector, and the component for the *other* axis is zero.

(Two arrows are "parallel" if they point in the same direction. Two arrows are "anti-parallel" if they point in opposite directions.)

$\vec{v}_0$ is parallel to the x-axis, so we can use the above rule to break $\vec{v}_0$ into components. $\vec{v}_0$ points right, which is our positive x-direction, so v_x is positive. v_x has the same magnitude as the overall vector, so $v_x = +v_0$. And the other component, v_{0y}, is zero.

Our work with the kinematics equations on the previous page provides us the answer to the problem.

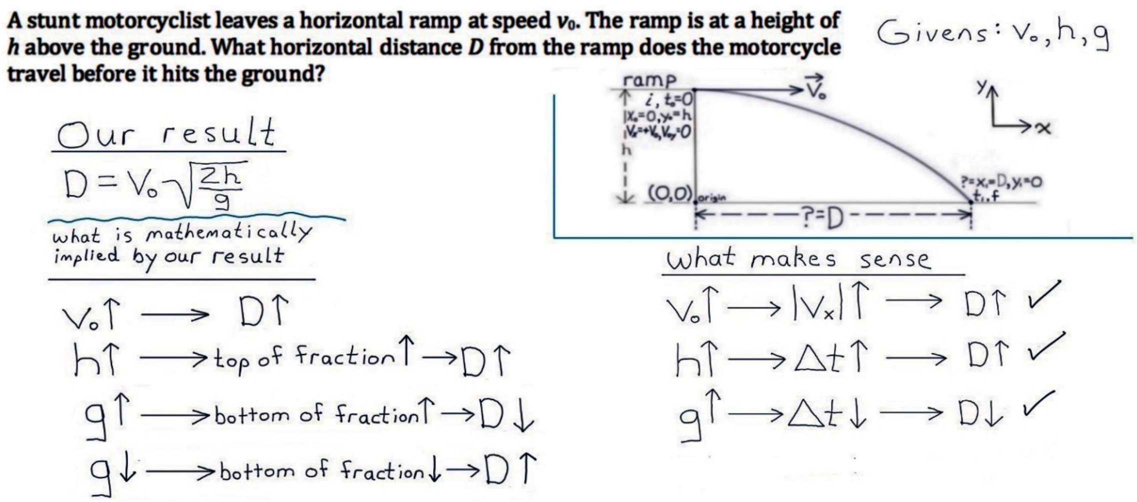


A symbolic answer should not include units.

For a symbolic problem, only symbols on our list of givens should be included in our final answer. Our list of given symbols is "Givens: v_0, h, g".

Our answer includes the symbols v_0, h, and g. All of those symbols are contained in our list of givens, so our answer is acceptable.

Do our results make sense?



The equation $D=v_0\sqrt{\frac{2h}{g}}$ has the following implications:

If we increase v_0 (holding the other givens constant), D will increase.
If we increase h (holding the other givens constant), D will increase.
If we increase g (holding the other givens constant), D will *decrease*.
Do these results make sense?

v_0 represents the motorcycle's initial speed. Since the motorcycle jumps off a horizontal ramp, the motorcycle's horizontal speed throughout the jump will equal v_0. Suppose we hold the height of the ramp constant. If we increase the initial speed (v_0), that will increase the motorcycle's horizontal speed ($|v_x|$) throughout the jump. Increasing the horizontal speed will increase the horizontal distance (D) that the motorcycle travels. So, yes, it does make sense that increasing v_0 will increase D.

h represents the height of the ramp. Suppose we hold the initial speed constant. If we increase the height (h) of the ramp that the motorcycle is jumping off of, that will increase the time that elapses (Δt) before the motorcycle hits the ground. Increasing the time elapsed will increase the horizontal distance (D) traveled. So, yes, it does make sense that increasing h will increase D.

Although we usually think of g as a physical constant, we can change g by redoing the experiment on a different planet. Suppose we redo the experiment on a planet with stronger gravity. This will result in a larger magnitude of acceleration due to gravity, which means a larger g. And suppose we hold the initial speed and height of the ramp constant. On a planet with stronger gravity (which results in a larger g), it will take less time (Δt) for the motorcycle to be pulled down to the ground. If the motorcycle spends less time flying above the ground, it will travel a shorter horizontal distance (D). So, yes, it does make sense that increasing g will decrease D.

So, our result $D=v_0\sqrt{\frac{2h}{g}}$ tells us that, if you want to make a long motorcycle jump, there are three ways to jump further: jump with greater speed (larger v_0), jump off a higher ramp (larger h), or jump off a ramp on the moon (smaller g).

Note: You could also think about whether our result that $\Delta t = \sqrt{\frac{2h}{g}}$ makes sense. For example, v_0 does not appear in the result for Δt; so, our result for Δt implies that an increase in v_0 will have *no effect* on Δt. Can you explain why that makes sense?

Recap

To succeed with physics, think in terms of *components*. We had to break the motorcycle's initial velocity into components. Because the motorcycle jumps off a horizontal ramp, the motorcycle's initial velocity is *horizontal*. Because the initial velocity is horizontal, the initial *vertical* velocity (v_{0y}) is *zero*, and the magnitude of the *horizontal* velocity throughout the jump (the magnitude of v_x), is equal to the motorcycle's initial speed (v_0).

For a symbolic problem, you should *write down* a list of the given symbols. Symbols that are explicitly mentioned in the problem, like v_0 and h, are treated as givens.
Exception: D is *not* treated as a given, even though the symbol D is mentioned in the problem, because D is what the problem is asking us for.
Symbols that are not explicitly mentioned in the problem, like Δt, are *not* treated as givens. Exception: g is treated as a given, even though the symbol g is not mentioned in the problem, because g stands for a known physical constant, 9.8 m/s^2.
Why is it helpful for us to write down this list of givens? The symbols on our list of givens are treated as "knowns"; symbols that are not on our list of givens are treated as "unknowns". Also, only symbols that are on our list of givens should be included in our final answer.

Because this problem is symbolic, we represented the magnitude of a_y with the symbol g, rather than with the number 9.8 m/s^2. The symbol g, written without an arrow on top, stands only for the *magnitude* of the acceleration due to gravity (g = 9.8 m/s^2). We chose *up* as our positive y-direction; so, when we write a_y, it's *crucial* for us to include a negative sign ($a_y = -g$) , to indicate that, for projectile motion, a_y points *downward*.

To get Δt by itself in the kinematics equation, we had to remove the ½ and the g from the left side of the equation. The most efficient way to do that is to multiply both sides of the equation by the *reciprocal* of ½, and by the reciprocal of g.

$$\frac{1}{2}g(\Delta t)^2 = h$$
$$\frac{1}{2}\cdot\frac{g}{1}(\Delta t)^2 = h$$
$$\frac{\cancel{2}}{1}\cdot\frac{1}{\cancel{g}}\cdot\frac{1}{\cancel{2}}\cdot\frac{\cancel{g}}{1}(\Delta t)^2 = \frac{h}{1}\cdot\frac{2}{1}\cdot\frac{1}{g}$$
$$(\Delta t)^2 = \frac{2h}{g}$$

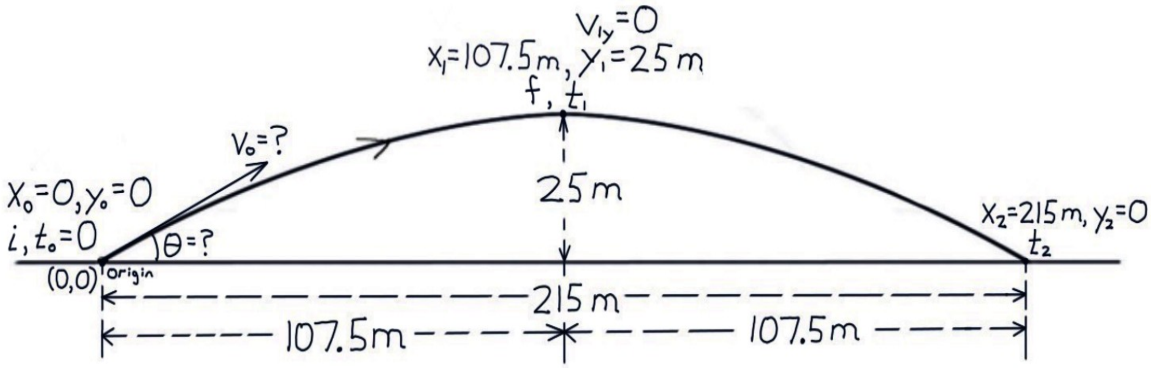
Build as much information as possible into your sketch. Draw a *large* sketch, so there will be ample room to include all pertinent information.
Use the information you've built into your sketch to create a projectile motion "setup". Then use the kinematics equations to solve the problem.

Video (4)

Part (a)

A golf ball is hit from the ground into the air. The ball reaches a maximum height of 25 m, and travels a horizontal distance of 215 m before it hits the ground.

- **(a) Calculate the initial speed and direction with which the ball was hit.**
- **(b) How long was the ball in the air?**
- **(c) What is the smallest value of the ball's speed over its entire trajectory?**



In projectile motion, the vertical velocity at the peak of the trajectory is zero. We use t_1, not t_2, as our "final" point, because we know the vertical velocity at t_1.

need

$$x_f = x_i + \underset{\text{need}}{v_x}\,\Delta t$$

$$107.5 = 0 + v_x \Delta t_{01}$$

$$107.5 = v_x \Delta t_{01}$$

$$107.5 = v_x (2.255)$$

$$\frac{107.5}{2.255} = \frac{v_x \cancel{(2.255)}}{\cancel{2.255}}$$

$$v_x = +47.7\ \tfrac{m}{s}$$

$\Delta t,\ y_i,\ y_f,\ \underset{\text{need}}{v_{iy}},\ v_{fy},\ a_y$

$\Delta t_{01},\ 0,\ 25\,m,\ v_{oy},\ 0,\ -9.8\ \tfrac{m}{s^2}$

$+22.1\ \tfrac{m}{s}$

$$v_{fy}^2 = v_{iy}^2 + 2a_y(y_f - y_i)$$

$$0^2 = v_{oy}^2 + 2(-9.8)(25-0)$$

$$0 = v_{oy}^2 - 490$$

$$+490 \qquad +490$$

$$490 = v_{oy}^2$$

$$v_{oy} = \sqrt{490}$$

$$v_{oy} = +22.1\ \tfrac{m}{s}$$

$$v_{fy} = v_{iy} + a_y \Delta t_{01}$$

$$0 = 22.1 + (-9.8)\Delta t_{01}$$

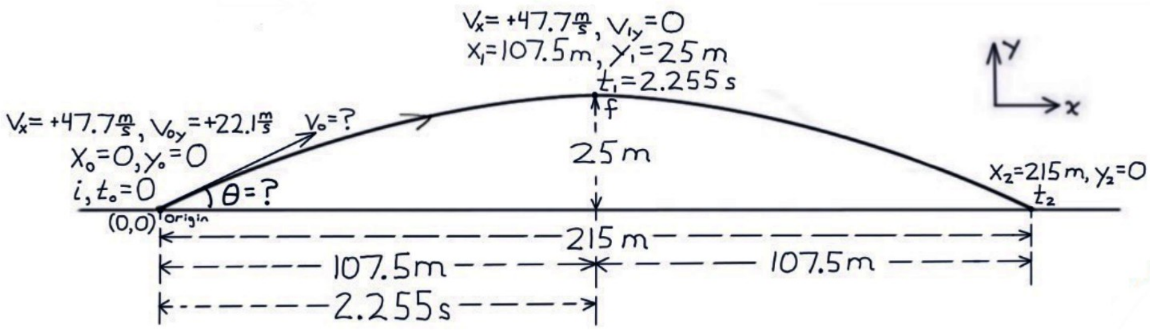
$$0 = 22.1 - 9.8\Delta t_{01}$$

$$+9.8\Delta t_{01} \qquad +9.8\Delta t_{01}$$

$$9.8\Delta t_{01} = 22.1$$

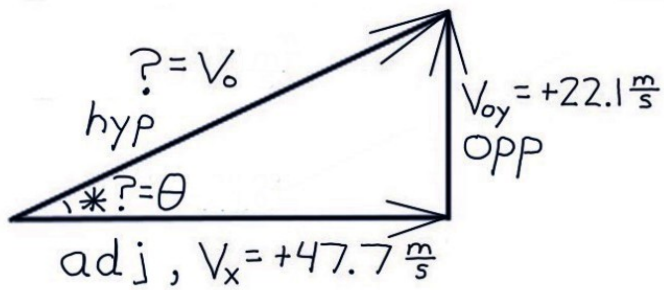
$$\frac{\cancel{9.8}\Delta t_{01}}{\cancel{9.8}} = \frac{22.1}{9.8}$$

$$\Delta t_{01} = 2.255\ s$$



As shown on the previous page, we used the kinematics equations to find the components of the velocity at t_0. Now, we use the components to find the magnitude and direction of the overall velocity vector at t_0. Notice that the direction of the overall vector can be represented by the angle θ.

(a) Calculate the initial speed and direction with which the ball was hit.



Pythagorean theorem

$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{other leg})^2$

$$V_0^2 = 47.7^2 + 22.1^2$$
$$V_0^2 = 2764$$
$$V_0 = \sqrt{2764}$$
$$V_0 = 52.6\frac{m}{s}$$

SOH CAH TOA

$$\tan\theta = \frac{opp}{adj}$$
$$\tan\theta = \frac{22.1}{47.7}$$
$$\tan\theta = .463$$
$$\theta = \tan^{-1}.463$$
$$\theta = 24.8^\circ$$

Answer to part (a)

> The ball is hit with an initial speed of $53\frac{m}{s}$, and an initial direction at an angle of 25° above the horizontal.

Always *check* to make sure that you've answered *all* parts of the question. Many students would fall into the trap of saying that the answer to part (a) is "53 m/s", forgetting that the question also asks for the *direction* of the velocity.

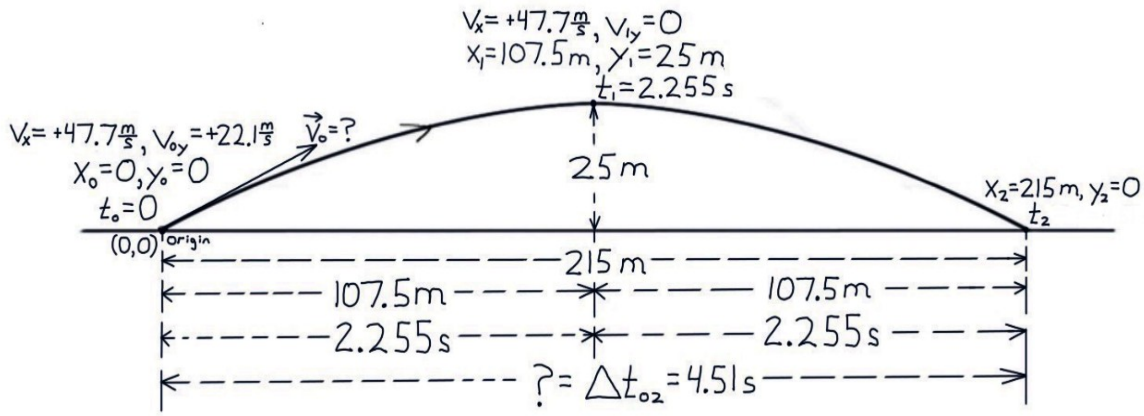
part (b)

A golf ball is hit from the ground into the air. The ball reaches a maximum height of 25 m, and travels a horizontal distance of 215 m before it hits the ground.

(a) Calculate the initial speed and direction with which the ball was hit.

(b) How long was the ball in the air?

(c) What is the smallest value of the ball's speed over its entire trajectory?



$$\Delta t_{02} = \Delta t_{01} + \Delta t_{12}$$
$$\Delta t_{02} = 2.255\,s + 2.255\,s$$
$$\Delta t_{02} = 4.51\,s$$

Answer to part (b)

The ball was in the air for 4.5 s.

Always try to use the exact right symbols, with the exact right subscripts:
Δt_{01} = the time that elapses between t_0 and t_1
Δt_{12} = the time that elapses between t_1 and t_2
Δt_{02} = the time that elapses between t_0 and t_2

Projectile motion is symmetric. So the time it takes for the object to rise from a particular height to the peak will be the same as the time it takes for the ball to fall back down from the peak to that particular height. So the time it takes for the ball to rise from the ground to the peak will be the same as the time it takes for the ball to fall back down from the peak to the ground. So the time that elapses between t_0 and t_2 will be double the 2.255 s that elapses between t_0 and t_1.

Also, the horizontal distance that the object travels as it moves from a particular height up to the peak, is equal to the horizontal distance that the object travels as it moves from the peak back down to that particular height. So the horizontal distance that the ball travels as it moves from a the ground to the peak, is equal to the horizontal distance that the ball travels as it moves from the peak back down to the ground. That's how we knew in part (a) that the horizontal distance traveled between t_0 and t_1 is half of the 215 m horizontal distance traveled between t_0 and t_2.

Always *check* to make sure that you've "answered the right question". Many students would fall into the trap of saying that the answer for part (b) is 2.255 s. To avoid that type of trap, make it a habit to *build the question into the sketch*, as shown above.

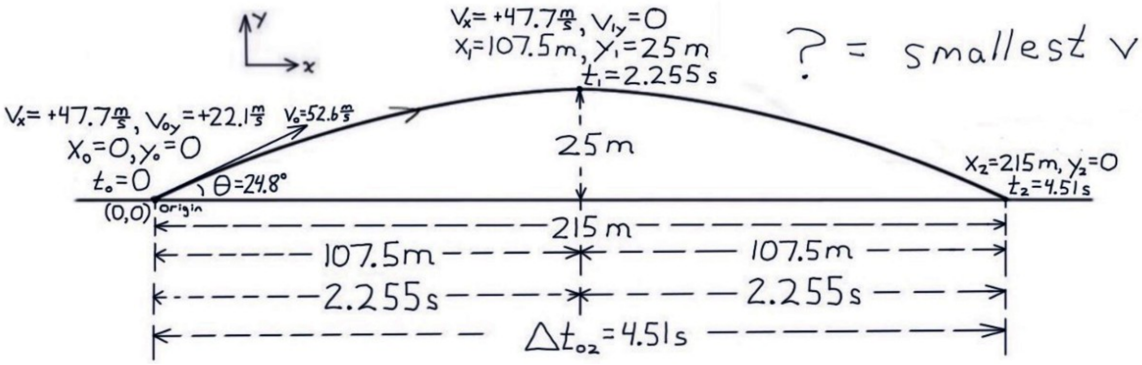
part (c)

A golf ball is hit from the ground into the air. The ball reaches a maximum height of 25 m, and travels a horizontal distance of 215 m before it hits the ground.

(a) Calculate the initial speed and direction with which the ball was hit.

(b) How long was the ball in the air?

(c) What is the smallest value of the ball's speed over its entire trajectory?



horizontal speed is constant

vertical speed is changing

So the smallest overall speed will occur when the ball experiences its smallest vertical speed. The smallest vertical speed is $V_{1y} = 0$, at the peak of the trajectory. At t_1, the overall speed equals the horizontal speed, $47.7\frac{m}{s}$. So the minimum speed is $v_1 = 47.7\frac{m}{s}$.

To succeed with physics, think in terms of components!

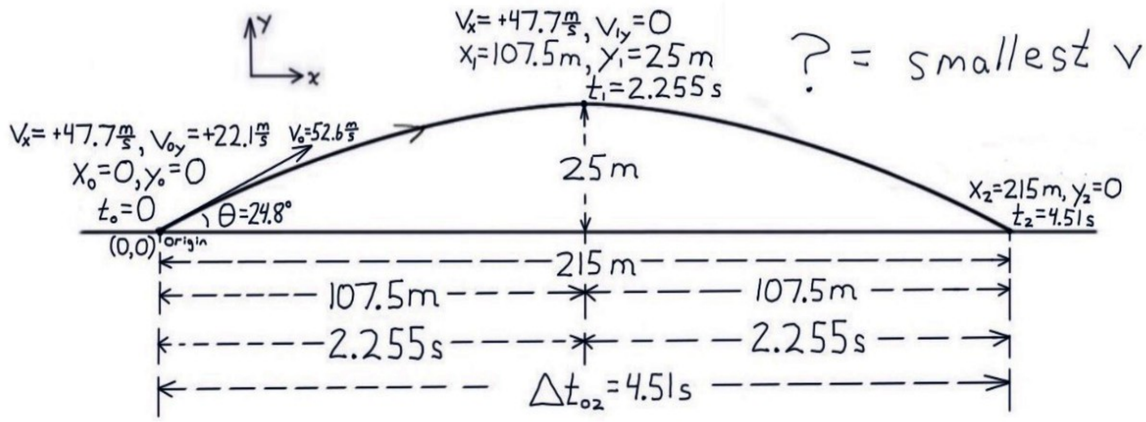
In projectile motion, the vertical velocity is changing, but the *horizontal* velocity is *constant*. So the smallest speed will occur at the point with the smallest *vertical* speed. This occurs at the peak of the trajectory, t_1, when the vertical velocity is zero. At t_1, the overall speed equals the horizontal speed, 47.7 m/s.

If you were unable to figure out the answer for part (c), then you need to make a stronger effort to *think in terms of components*.

Answer for part (c)

$$\boxed{\text{The smallest value of the ball's speed over its whole trajectory is } 48\tfrac{m}{s}.}$$

Do our results make sense?



Remember that 1 meter is about 1 yard. And remember that a football field is 100 yards long.

The problem told us that the ball traveled a horizontal distance of 215 m, which is about 215 yards, or about two football fields. That's a reasonable driving distance for a golfer.

We found that it takes the ball about 4 and a half seconds to travel that distance. That's a reasonable hang time for a golf drive.

The ball reaches a height of 25 meters, which is about 25 yards, ¼ of a football field.

We found that the ball was hit with an initial speed of 52.6 m/s.

1 m/s is roughly 2 miles per hour, so the ball was hit with an initial speed of about 100 mi/hr. That means that the club head was moving at about 100 mi/hr when it hit the ball. That's a reasonable club head speed for a golfer hitting a drive.

Note: In reality, a golf ball experiences significant drag forces during its flight. And as a result of the ball's spin and "dimpled" surface, a golf ball will also experience significant lift forces. Therefore, the pure "projectile motion" analysis in this problem is not a very accurate model of how an actual golf ball will behave in flight. Pure projectile motion is symmetric; but the motion of an actual golf ball in flight is not symmetric.

Recap

In projectile motion, the *vertical* velocity at the *peak* of the trajectory is *zero*. So we were able to substitute *zero* for v_{1y}, the vertical velocity at t_1.

The "initial" and "final" points are defined as the points that we plan to substitute into our kinematics equations. We know the position coordinates at both t_1 and t_2. Because we know the vertical velocity at t_1, but we don't know the vertical velocity at t_2, we chose to use t_1, not t_2, as the "final" point that we will plug into our kinematics equation. Don't assume that you should always choose he beginning and end of the projectile motion path as your "initial" and "final" points.

We used the components of the velocity at t_0 to find the magnitude and direction of the overall velocity at t_0. We used the Pythagorean theorem to find the magnitude of the overall velocity vector.

The problem also asks for the direction of the overall velocity vector. The *direction* of the overall velocity vector can be described by an *angle*. We used a SOH CAH TOA equation to find the angle formed by the velocity vector. To solve this equation, we had to use the *inverse* tangent function.

Projectile motion is symmetric. So we were able to calculate that the horizontal distance traveled while the ball rises from the ground to to peak of the trajectory was *half* of the *total* horizontal distance traveled before the ball hits the ground.

And we were able to calculate that the total time elapsed while the ball is in the air is double the time that elapses while the ball rises from the ground to the peak.

In projectile motion, the vertical velocity is changing, but the horizontal velocity is constant. We found that the ball's horizontal velocity is 47.7 m/s. That value applies at every point along its trajectory. So we use the same symbol, v_x, for the horizontal velocity at every point along the trajectory.

We found that the ball's vertical velocity at t_0 is +22.1 m/s. So, the ball's vertical velocity *changes* from +22.1 m/s at t_0, to zero at t_1. So we use different symbols (v_{0y} and v_{1y}) for the vertical velocity at t_0, and the vertical velocity at t_1.

To succeed with physics, think in terms of *components*.

The ball's *horizontal* speed is constant; so, to find the point with the smallest *overall* speed, we had to find the point with the smallest *vertical* speed.

For a projectile motion problem, **build as much kinematics information as possible into your sketch**. Build all the information provided in the problem into your sketch; and whenever you figure out *new* information, build that into your sketch.

When possible, build the question into the sketch. Building the question into the sketch will help you to avoid the trap of saying that the answer to part (b) is 2.255 s.

Draw a *large* sketch, so that you'll have ample room to clearly include all the information you need in your sketch.